HARDENING OF AN ELASTOPLASTIC BODY

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The purpose of the experimental investigations whose results are described here is to study plastic states in which loading occurs in some directions and unloading in others, and also to verify the applicability of the plastic strain schemes proposed in [1, 2] for these states. The existence of an angular point on the loading surface is detected. The influence of the loading path on the development of this point is investigated. A load of biaxial tension type was produced in a 45KhN steel-pipe specimen subjected to internal pressure and an axial force. The ratio between the principal stresses hence varied, but the principal directions of the stress tensor remained fixed in the body of the specimen.

Hollow cylinders with a $\delta = 1$ mm wall thickness and $R = 15$ mm average radius in the working section were used as test specimens. The technology for fabricating the specimens which was presented in [3] was used to assure equal wall thickness. The difference in the wall thickness did not exceed ± 0.01 mm both along the length of the working section and along the circumference. The specimens were heat treated at a 720°C temperature in a 10^{-4} mm Hg vacuum with 2.5-h holding and a 5-h stay in the furnace.

The strains were measured by strain gauges with clock-type displays: the longitudinal strains on a 100-mm base by displays with a 0.01-mm scale division and the transverse, by a micron display. The radial strain was determined by using the hypothesis about the elastic change in volume, and the radial stress was taken as zero. To eliminate the influence of creep, the strain measurements were conducted after a 5-min holding at each step of the loading. The Young's modulus under uniaxial tension is $E = 21{,}600$ kg/mm^2, the Poisson ratio is $\nu = 0.23$, and the flow area was observed during the passage to plastic strain.

Let us examine the stress state of an element with fixed principal directions of the stress tensor $\sigma_1 \geq \sigma_2 \geq \sigma_3$. The principal tangential stresses $2T = \sigma_1 - \sigma_3$, $2T_{12} = \sigma_1 - \sigma_2$, $2T_{23} = \sigma_2 - \sigma_3$ are connected by the Lode parameter μ_0:

$$T\mu_0 = T_{23} - T_{12}.$$

The parameter $\mu_{\Delta\sigma}$ is introduced for the increments in these tangential stresses

$$\Delta T \mu_{\Delta\sigma} = \Delta T_{23} - \Delta T_{12}.$$

and the parameter $\mu^p_{\Delta\varepsilon}$ for increments in the plastic strain $\Delta\varepsilon^p_\alpha$ ($\alpha = 1, 2, 3$):

$$\mu^p_{\Delta\varepsilon} = \frac{2\Delta\varepsilon^p_2 - \Delta\varepsilon^p_1 - \Delta\varepsilon^p_3}{\Delta\varepsilon^p_1 - \Delta\varepsilon^p_3}. \tag{1}$$

If $\Delta\varepsilon^p_1 \geq \Delta\varepsilon^p_3 \geq \Delta\varepsilon^p_2$, then it is convenient to calculate $\mu^p_{\Delta\varepsilon}$ by means of the formula

$$\mu^p_{\Delta\varepsilon} = \frac{2\Delta\varepsilon^p_3 - \Delta\varepsilon^p_1 - \Delta\varepsilon^p_2}{\Delta\varepsilon^p_1 - \Delta\varepsilon^p_2}. \tag{2}$$

Following [4], let us introduce the stress intensity σ_i, the shear strain intensity ε_i, and the angle of the stress state ω_σ:

$$\mu_\sigma = \sqrt{3}\,\mathrm{ctg}\,(\omega_\sigma + \pi/3). \tag{3}$$

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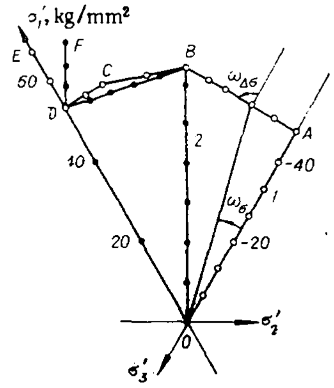

Fig. 1

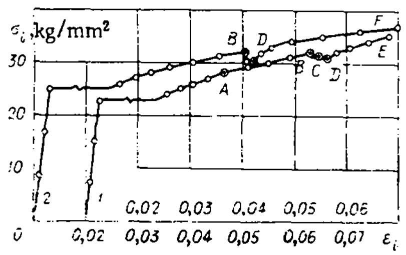

Fig. 2

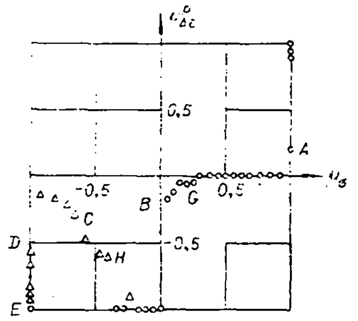

Fig. 3

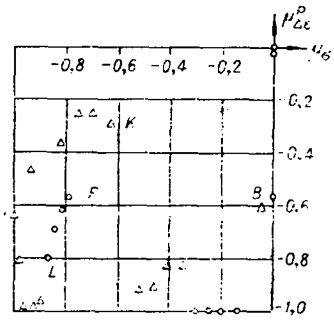

Fig. 4

TABLE 1

Tangential stress	T	T_{12}	T_{23}	$\frac{T+T_{12}}{2}$	$\frac{T+T_{23}}{2}$
$\omega_{\Delta\sigma}$	$\frac{2\pi}{3}$	π	$-\frac{2\pi}{3}$	$\frac{5\pi}{6}$	$\frac{\pi}{2}$

TABLE 2

Section	OA	AB	BC	CD	DE	OB	BD	DF
$\omega_{\Delta\sigma}$	0	$\frac{\pi}{2}$	134°	$\frac{5\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{6}$	139°	$\frac{\pi}{6}$

By analogy with (3), the angles $\omega_{\Delta\sigma}$ and $\omega^{p}_{\Delta\varepsilon}$ are considered for $\mu_{\Delta\sigma}$ and $\mu^{p}_{\Delta\varepsilon}$. The angles ω_{σ} and $\omega_{\Delta\sigma}$ have an explicit meaning on the deviator plane (Fig. 1). The directions of the lines of constant tangential stress on the deviator plane, which are henceforth designated by the lines T, T_{12}, T_{23}, $(T+T_{12})/2$, $(T+T_{23})/2$, can be set by using the angle $\omega_{\Delta\sigma}$. These directions are indicated in Table 1.

Loading trajectories on the deviator plane are presented in Fig. 1: OABCDE for specimen 1 and OBDF for specimen 2. The values of $\omega_{\Delta\sigma}$ for the loading sections are indicated in Table 2.

The dependences $\varepsilon_i(\sigma_i)$ for the loading trajectories presented in Fig. 1 are constructed in Fig. 2. These dependences disclose a qualitative contradiction with the classical deduction of deformation theory in that the character of the loading path does not influence the dependence $\varepsilon_i(\sigma_i)$. It is seen that the Mises surface is not a loading surface. Similar conclusions have been made in [5], where loading trajectories with constant stress intensities σ_i were studied. These effects are explained in [1, 2] by the presence of states such that plastic strain occurs in some directions and unloading in others, i.e., anisotropies of the plastic state.

Let us examine the nature of the plastic strain in more detail by using the parameter $\mu^{p}_{\Delta\varepsilon}$. The dependences of $\mu^{p}_{\Delta\varepsilon}$ on μ_{σ} are presented in Figs. 3 and 4 for specimens 1 and 2, respectively, where the circles

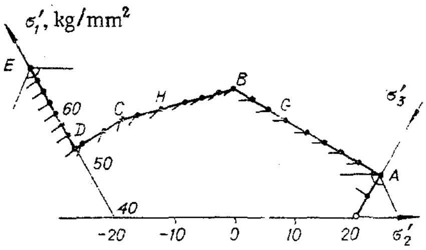

Fig. 5

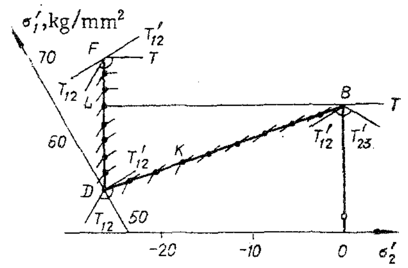

Fig. 6

TABLE 3

Tangential stress		T	T_{12}	T_{23}	$\frac{T+T_{12}}{2}$	$\frac{T+T_{23}}{2}$
$\mu^p_{\Delta\varepsilon}$	(1)	0	−3	3	−1	1
	(2)	—	0	—	−1	—

correspond to values of $\mu^p_{\Delta\varepsilon}$ according to (1) and the triangles, according to (2). The degree of specimen isotropy was clarified by determining the values of $\mu^p_{\Delta\varepsilon}$ for specimen 1 and of $\mu^p_{\Delta\varepsilon}$ for specimen 2 on the proportional loading section (these quantities vanish in the case of initial specimen isotropy). It turns out that these values are within the limits 0–0.1 in the first case and within 0–0.02 in the second, which indicates the presence of an initial anisotropy of the specimen properties. However, these deviations are small compared to those disclosed at the breakpoint of the loading trajectory at the points A, B, D and also at the point H in Fig. 3 and the points B and K in Fig. 4; i.e., the vector of the plastic strain increments abruptly changes its direction depending on the direction of the stress increment vector. In loading surface terminology, this means the existence of an angular point at which the angle $\omega^p_{\Delta\varepsilon}$ is undefined and depends on the angle $\omega_{\Delta\sigma}$. By knowing the extreme values of $\omega^p_{\Delta\varepsilon}$ and by using the hypothesis about normality of the plastic strain rate vector to the loading surface, we estimate the nature of the angular point exactly as has been done in [6].

The loading trajectory for specimen 1 is shown on the deviator plane in Fig. 5, where directions perpendicular to the plastic strain increment vector are presented (the circles correspond to the time the flow area appears). The change in this direction at the point A indicates that the projection of loading surface on the deviator plane adjoins the point A on the left at the angle $\omega_{\Delta\sigma}=2\pi/3$. Because of symmetry of the loading surface relative to the loading section OA, it follows that it adjoins the point A on the right with the angle $\omega_{\Delta\sigma}=-2\pi/3$. Therefore, the projection of the loading surface on the deviator plane contains an angular point formed by the lines T and T_{23} at the point A (see Table 1).

It is analogously clarified that this projection of the loading surface adjoins the point B on the left at the angle $\omega_{\Delta\sigma}=5\pi/6$, i.e., the line $(T+T_{12})/2$ is formed, and the point C at the angle $\omega_{\Delta\sigma}=\pi$, i.e., the line T_{12} is formed. It is seen from the analysis performed for the loading surface that the nature of the angular point depends essentially on the kind of stress state μ_σ.

A mechanical scheme for inelastic deformation of a body is proposed in [1, 2], in which the plastic strain is represented as a sequence of shears over the areas of operation of the principal tangential stresses T, T_{12}, T_{23}, called the slip areas T, T_{12}, T_{23}. By using this scheme, the results in Fig. 3 and Table 3, where the correspondence between the slip areas and the values of $\mu^p_{\Delta\varepsilon}$ in the form of (1) and (2) is presented, can permit recognition of along which areas shears are realized. Thus, for instance, shears over the slip areas T (the loading surface is formed by the line T) are realized on the loading section AG (see Fig. 3). In this case it is convenient to represent the arbitrary additional loading as [1]

$$\begin{pmatrix}\Delta\sigma_1 & 0 & 0\\ 0 & \Delta\sigma_2 & 0\\ 0 & 0 & \Delta\sigma_3\end{pmatrix}=\Delta\sigma_n\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}+\Delta T\begin{pmatrix}1&0&0\\0&0&0\\0&0&-1\end{pmatrix}+\Delta\sigma_2'\begin{pmatrix}0&0&0\\0&1&0\\0&0&0\end{pmatrix}, \tag{4}$$

where $2\Delta\sigma_n=\Delta\sigma_1+\Delta\sigma_3$ and $\Delta\sigma_2'=\Delta\sigma_2-\Delta\sigma_n$. Only the second tensor on the right side in (4) is related to the plastic strain.

Shear occurs on the slip areas T_{12} (the loading surface is formed by the lines T_{12}) on the loading section CD. In this case it is convenient to carry out the expansion (4) by means of ΔT_{12}.

It can analogously be concluded that shears over the slip areas $T'_{12} = (T + T_{12})/2$ are realized on the loading section BH, and over the slip areas T_{12} and T'_{12} simultaneously on the HC and DE sections. Unloading with hardening hence sets in along the slip areas T; i.e., these areas do not contribute to the plastic strain. Therefore, plastic shears are realized along some slip areas on the loading sections considered and unloading with hardening along others. The stress intensity σ_i hence diminishes.

Shown in Fig. 6 is the nature of the plastic strain in terms of the loading surface for specimen 2 (the circle corresponds to the time the flow area appears). The deformation scheme [1, 2], the results in Fig. 4, and considerations about the symmetry of the loading surface relative to the loading section OB were hence used. For example, the point B is an angular point of the loading surface formed by the lines $T'_{12} = (T + T_{12})/2$, $T'_{23} = (T + T_{23})/2$. The loading surface is formed by the lines T'_{12} on the segment BK (unloading with hardening occurs along the slip areas T, T'_{23}). Then the loading K again becomes an angular point of the loading surface (shear along the slip areas T_{12} is realized) formed by the lines T'_{12} and T_{12}, while the loading surface is formed by the lines T_{12} and T at the point L (shear along the slip areas T is realized).

Therefore, the development of the angular point detected on the loading surface is described completely satisfactorily by the mechanical scheme [1, 2], supplemented by the slip areas T'_{12}, T'_{23}. This permits making the following deductions: 1) the loading point is an angular point on the loading surface; 2) hardening is not isotropic but depends essentially on the kind of stress state μ_σ; 3) this hardening is described completely satisfactorily by the mechanical plastic strain scheme [1, 2] supplemented by the slip areas T'_{12}, T'_{23}.

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